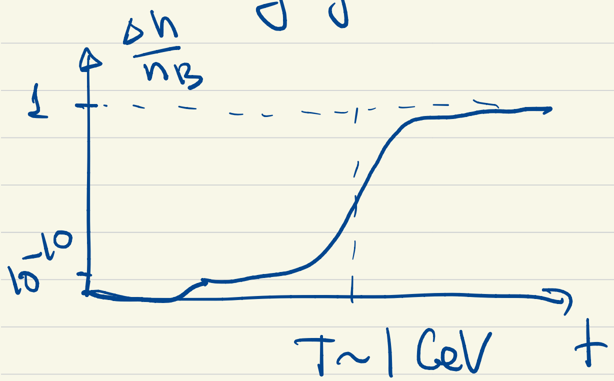
Lecture 10

- Review
- Dark Matter candidates (continued)
- Inhomogeneous universe
  → Newtonian theory for cosmological perturbations

Review

→ Baryogenesis



· Sakharov's conditions:
→ violation of B
→ violation of C and CP
→ no thermal equilibrium

→ Dark matter

- evidence for DM (rotational curves, measurements of total matter contribution (BBN, CMB...), Bullet cluster, lensing.
- properties

$m_f \gtrsim 500\,\mathrm{eV}$ (excludes ν)

$m_B \gtrsim 10^{-20}\,\mathrm{eV}$

DM candidates

→ QCD Axion

(is motivated by strong-CP problem – absence of CP violation in QCD). One explanation is to add Axion (light bosonic particle $P: a \to -a$)

Experimental constraints on mass

$$10^{-4}\,eV \lesssim m_a \lesssim 10^{-2}\,eV$$

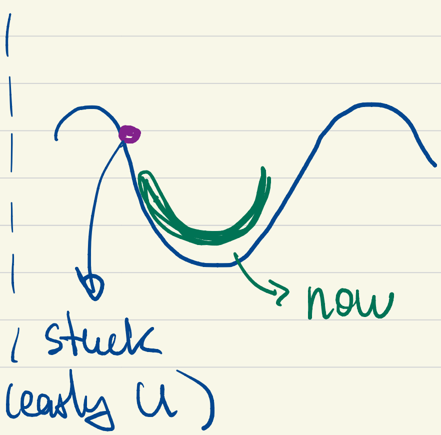
Axion can also be DM particle if $m_a \lesssim 10^{-5}\,eV$

$$V_a\Big|_{T=0} \simeq \underbrace{\Lambda^4_{QCD}}_{(\mathrm{GeV})^4} \cos\frac{a(x,t)}{\underbrace{f_a}_{\text{const}}}$$

$$m_a^2 \simeq \frac{\Lambda^4_{QCD}}{f_a^2}, \quad \rho_a \sim m_a^2 f_a^2 \cos\theta \sim m_a^2 f_a^2 \theta^2$$

$$\theta = \frac{a}{f_a}$$

$$n_a \sim \frac{\rho_a}{m_a}$$



$$\ddot a + \underbrace{3H\dot a}_{\text{Hubble friction term}} + m_a^2 a = 0$$

$H \gg m_a$ Axion is like C.C.

$H \ll m_a$ Axion is matter ($w=0$, $p=0$)

transition to oscillations happens

$$H(t_{osc.}) \sim m_a$$

[In reality Axion mass depends on T if $T > 1$ GeV]

$$\Omega_a \simeq \frac{10^{-6}\,eV}{m_a}\,\theta_i^2$$

• Ignore $m(T) \to$ const, estimate Ω_a:

$$H = \frac{T^2}{M_*} \longrightarrow n_a\Big|_{\underset{t_{osc}}{i}} = \frac{T_{osc}^2}{M_*} f_a^2 \theta_i^2$$

$$n_{a,0} \simeq n_a^i \left(\frac{a_i}{a_0}\right)^3 \simeq n_a^i \left(\frac{T_0}{T_{osc}}\right)^3 \simeq$$

$$\simeq T_0^3 \frac{f_a^2 \theta_i^2}{M_{pl} T_{osc}}$$

$$\rho_{a,0} \simeq m_a T_0^3 \frac{f_a^2}{M_{pl} T_{osc}} \theta_i^2 \simeq T_0^3 \frac{\Lambda_{QCD}^4 \theta_i^2}{M_{pl}^{3/2} m_a^{3/2}}$$

$m_a \sim 10^{-5} \div 10^{-6}$ eV get $\Omega_a = \Omega_{DM}$ for $\theta_i \sim 1$ | θ_i in observed universe can be "antropically" small.

→ Axion-like particles

$V(a) \simeq \Lambda_a^4 \cos \frac{a}{f_a}$ (can be much lighter)

Weakly Interacting Massive Particle (WIMP)

"Weakly – interacts with EW force" gives the right abundance for a large class of models

- Production is by freeze-out mechanism

in thermal equil. initially

$$X + \bar{X} \longleftrightarrow SM$$

$$X \not\rightarrow SM$$

$$\Gamma_X = \langle \sigma n v \rangle \sim H$$

$\downarrow$ freeze out

$$\sigma \sim \frac{\sigma_0}{v}$$

$$n_X = g_X \left(\frac{MT}{2\pi}\right)^{3/2} e^{-\frac{M}{T}}$$

$$\sigma_0 g_x \left(\frac{M T_\theta}{2\pi}\right)^{3/2} e^{-\frac{M}{T_\theta}} \sim \frac{T_\theta^2}{M_*}$$

$$T_\theta \simeq M / \log(\dots)$$

$$n_x(t_0) \simeq n_x(t_\theta) \cdot \frac{a^3(t_\theta)}{a^3(t_0)} \sim n_x(t_\theta) \frac{T_\gamma^3}{T_\theta^3}$$

$$\Omega_x = \frac{n_x(t_0) \cdot M_x}{\rho_c} \simeq 10^{-10} \frac{\mathrm{GeV}^2}{\sigma_0}.$$

$\cdot \log(\dots) \quad \sim \frac{T_\theta^2}{M_*}$

$$\left[\ \Omega_x \sim \frac{H_\theta M}{T_\theta^3} \text{ is } M\text{-independent} \right]$$

$$\sigma_W \sim 10^{-10} \left(\frac{E}{\mathrm{GeV}}\right)^2$$

Direct detection exp. excluded many models.

→ Primordial Black Holes

- from cosmological point of view can have a very wide range of masses, there are many detailed exp. constraints

- After LIGO discovery

Inhomogeneous Universe

Newtonian theory of perturbation grow (short distances and timescales)

- Of course U. is not homogeneous on short scales

$$\frac{\delta\rho}{\rho} \simeq 1$$

- We know from CMB observations

$$\frac{\delta\rho}{\rho} \simeq 10^{-5} \quad (\text{at CMB time})$$

- General idea is that perturbations grow with time
- We will use Newtonian theory for now:

$$\nabla^2 \varphi = 4\pi G \rho$$

grav. potential — density of "matter"

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

Euler equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla} \underline{\underline{p}} - \vec{\nabla}\varphi$$

It is basically Newton's 2nd law:

$$F = ma$$

$$F = F_{\text{pressure}} + F_{\text{grav}} =$$

$$= -\int dV (\nabla \underline{\underline{p}} + \rho \cdot \nabla \varphi)$$

acceleration also two pieces

$$\frac{d\vec{v}}{dt} = \underset{\text{naive acceleration}}{\frac{\partial \vec{v}}{\partial t}} + \underbrace{\frac{\partial \vec{v}}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial \vec{v}}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial \vec{v}}{\partial z}\cdot\frac{\partial z}{\partial t}}_{\text{due to motion}} =$$

$$= \frac{\partial v}{\partial t} + (v\cdot\vec{\nabla})\vec{v}$$

$$m = \int \rho \, dV$$

- Expand into background and perturbations

$$\rho(x,t) = \rho_0(t) + \delta\rho(t,x)$$

$$p(x,t) = p_0(t) + \delta p(t,x)$$

$$v(x,t) = v_0(t) + \delta v(t,x)$$

$$\varphi(x,t) = \varphi_0(t) + \delta\varphi(t,x)$$

- Next we linearize our equations

$$\nabla^2\delta\varphi = 4\pi G\delta\rho$$

$$\frac{\partial\delta\rho}{\partial t} + \nabla(\delta\rho V_0) + \nabla(\rho_0\delta v) = 0$$

$$\frac{\partial\delta v}{\partial t} + (V_0\nabla)\delta v + (\delta v\cdot\nabla)V_0 + \frac{1}{\rho_0}\nabla\delta \underline{\underline{p}} + \nabla\delta\varphi = 0$$

Growth in the static universe (expansion slows down the growth) $H=0 \to \vec{V_0}=0$

$$\nabla^2\delta\varphi = 4\pi G\delta\rho \qquad (1)$$

$$\frac{\partial\delta\rho}{\partial t} + \nabla(\rho_0\delta v) = 0 \qquad (2)$$

$$\frac{\partial\delta v}{\partial t} + \frac{1}{\rho_0}\nabla\delta \underline{\underline{p}} + \nabla\delta\varphi = 0 \qquad (3)$$

$$v_s^2 = \frac{\delta P}{\delta \rho}$$

(3) $$\frac{\partial \delta v}{\partial t} + \frac{v_s^2}{\rho_0}\nabla\delta\rho + \nabla\delta\varphi = 0$$

take time der. of (2)

$$\frac{\partial^2 \delta \rho}{\partial t^2} + \rho_0 \vec{\nabla}\left(\frac{\partial \delta v}{\partial t}\right) = 0$$

take grad. of (3)

$$\frac{\partial}{\partial t}\vec{\nabla}\cdot\delta\vec{v} + \frac{v_s^2}{\rho_0}\Delta\delta\rho + \Delta\delta\varphi = 0$$

use (1)

$$\frac{\partial^2 \delta\rho}{\partial t^2} - v_s^2 \Delta\delta\rho - \rho_0 4\pi G\delta\rho = 0$$

look for plain wave solutions:

$$\rho \sim e^{ikx - i\omega t}$$

$$\omega^2 = v_s^2 k^2 - 4\pi G\rho_0$$

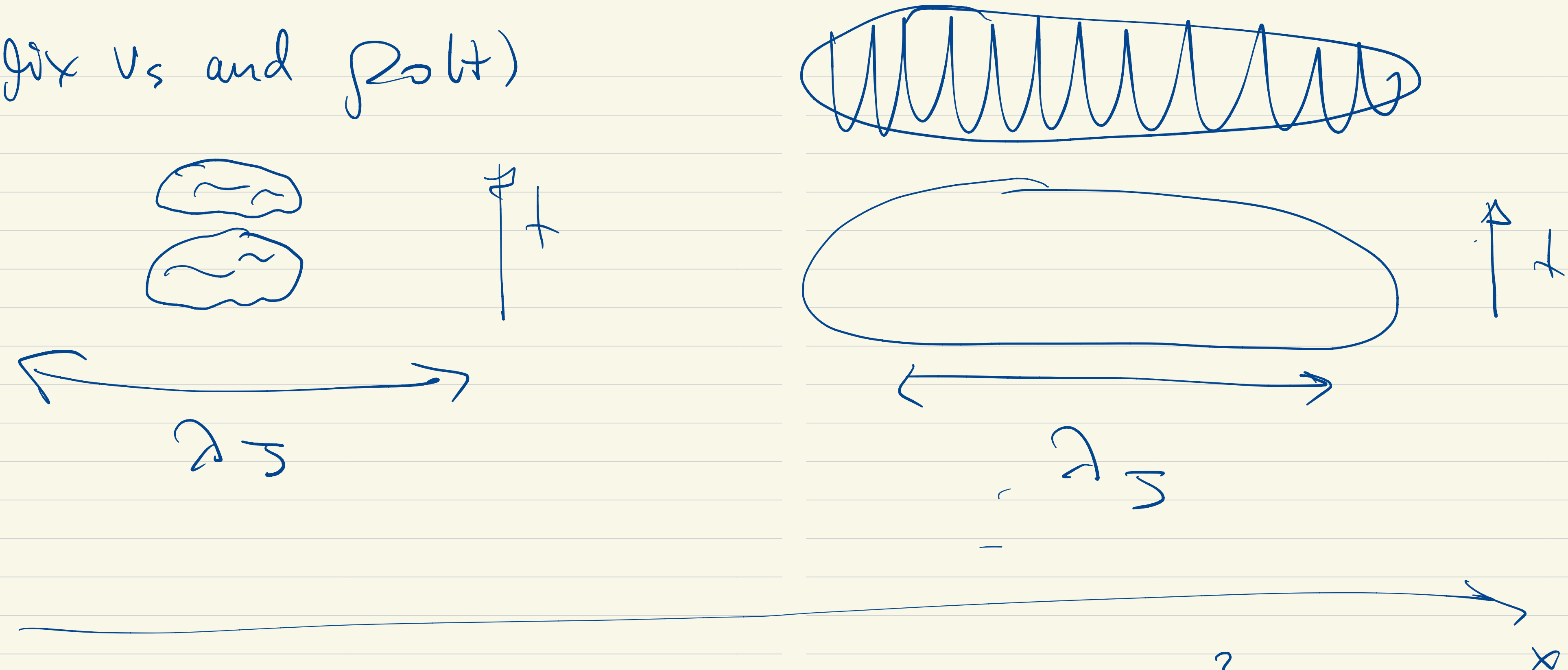
if $k < \frac{\sqrt{4\pi G\rho_0}}{v_s}$ ω is imaginary!

$\rho \sim e^{i\omega t}$ grows exponentially with time

The critical wavelength for which it happens is

Jeans instability $\longrightarrow$ $$\lambda_J = \frac{2\pi v_s}{\sqrt{4\pi G\rho_0}}$$

fix v_s and $\rho_0(t)$



$$M_J \simeq \lambda_J^3 \cdot \rho_0$$

rad. dom. $\delta\rho \sim \ln t$

matter dom. $\delta\rho \sim t^{2/3}$